So far we have only dealt with interest compounded annually. Interest can be compounded over any interval. Let's say you want to compound quarterly, you would apply one-fourth of the interest rate for four times the number of periods. For example, $1000 invested at 9% interest for 5 years compounded quarterly would be $1000\left(1+\frac{.09}{4}\right)^{4(5)}$ = 1000 $(1.0225)^{20}$ or 1560.51. That compares with 1000 $(1.09)^{5}$ or 1538.62 if the interest were compounded annually.

What would the balance be if the interest were compounded monthly? Daily? Every hour?

If you calculated it every second the answer would be $1568.31. Try using your calculator to find $1000(e^{.09})^{5}$. Do you see how your answers have been getting closer and closer to this number? The mathematical term for something we are getting closer and closer to is a limit. The constant e, euler's (pronounced oiler's) constant, is one of the most important universal constants, in the same category as π. It is approximately equal to 2.718281828.... . It can be defined as $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{n}$, the limit as n approaches infinity of $\left(1+\frac{1}{n}\right)^{n}$. Verify this for yourself by trying several increasingly large values for n. This means that we can calculate what the interest would be if compounding took place every instant. Find the value of $7000 at 13% interest for 38 years compounded annually. Then find the value if it were compounded instantaneously, that is, $7000e^{.13t}$.

The natural log, ***ln* x**, or $\log_e x$ is the inverse operation of e^x. That is, $\ln e^x = x$. If $1000e^t = 2000$, then $e^t = 2000/1000$, then $\ln(e^t) = \ln(2)$, then $t = \ln(2) = .69314…$. If $2000e^t = 10000$ then t = ________, if $2000e^t = 1000$ then t = ______. What does this answer mean?

Sometimes we write exponentials using euler's constant. This frequently makes calculations easier. So, instead of writing 1000 $(1.09)^t$, we might think of 1.09 as e^x. If $e^x = 1.09$ then $x = \ln(1.09)$ or about .086. This means that $1000(1.09)^t = 1000(e^{.086})^t = 1000(e^{.086t})$. Show how you would write $25000(1.12^t)$ = _____________, $15000(1.11^t)$ = ___________, $10000e^{.05t}$ = ___________.

So if you were to enter your paper tearing data into Excel and do an exponential regression, you would get $y = e^{.693147x}$. This is the same as $P=2^t$. The way you can tell is that $ln(2) = .693147$ and $e^{.693147} = 2$. If $e^{.693147} = 2$ then $(e^{.693147})^x = 2^x$.