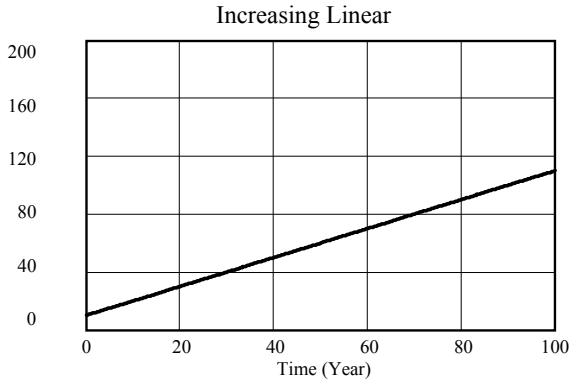
Increasing Linear or Arithmetic Sequence

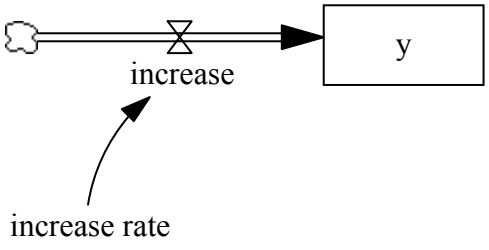
Adding a constant repeatedly
The increase is constant.
r is the common difference.

$$y = y_0 + rt, \quad r > 0$$

$$t = \frac{y - y_0}{r}$$

t	y
0	10
20	30
40	50
60	70
80	90
100	110





Exogenous Increase is controlled from outside the system. There is no feedback.

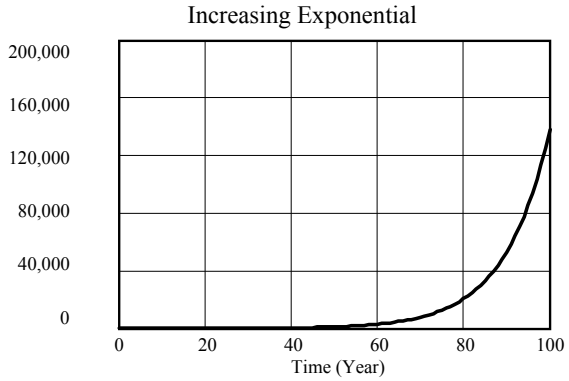
Increasing Exponential or Geometric Sequence

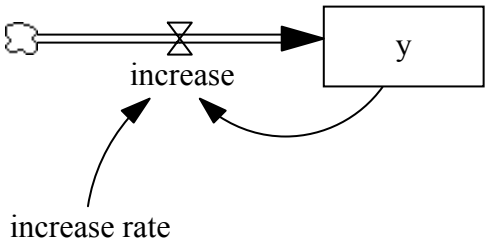
Multiplying by a constant repeatedly
The more you have, the more you get.
r is the common ratio.

$$y = y_0 \cdot r^t, \quad r > 1$$

$$t = \log_r\left(\frac{y}{y_0}\right) = \frac{\ln\left(\frac{y}{y_0}\right)}{\ln(r)} = \frac{\log\left(\frac{y}{y_0}\right)}{\log(r)}$$

t	y
0	10
20	67
40	453
60	3045
80	20484
100	137806





Endogenous Increase is controlled from inside the system. There is feedback.

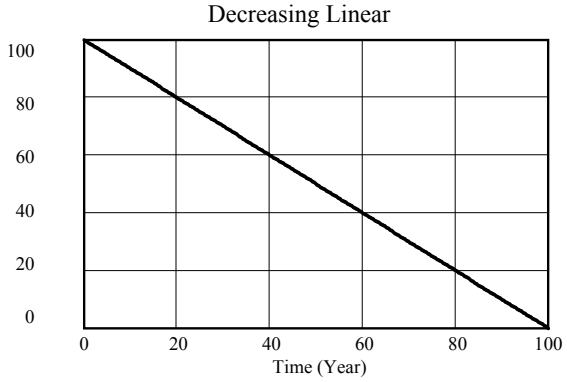
Decreasing Linear

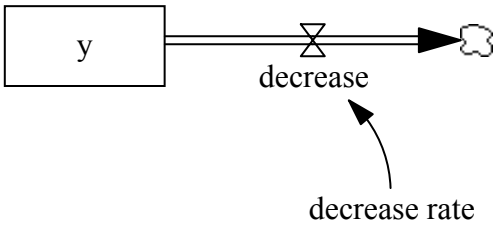
Adding a negative constant repeatedly
The decrease is constant.
r is the common difference.

$$y = y_0 + rt, \quad r < 0$$

$$t = \frac{y - y_0}{r}$$

t	y
0	100
20	80
40	60
60	40
80	20
100	0





Exogenous Decrease is controlled from outside the system. There is no feedback.

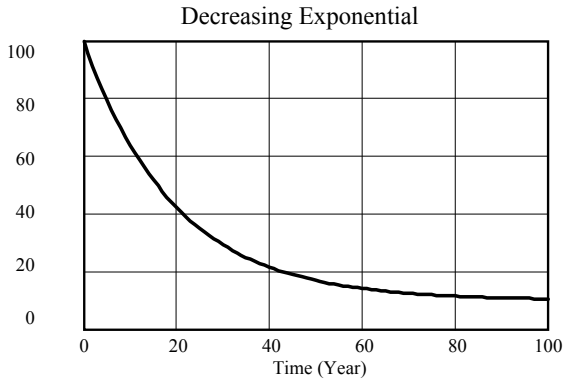
Decreasing Exponential

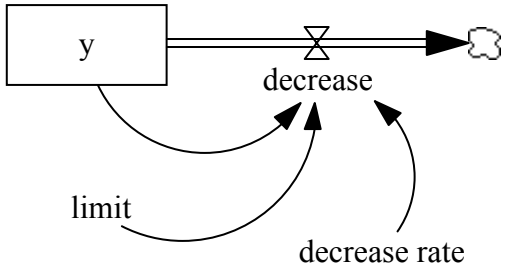
Multiply by a fractional constant repeatedly
The more you have, the more you lose.
r is the common ratio.

$$y = y_f + (y_0 - y_f) \cdot r^t, \quad r < 1$$

$$t = \log_r\left(\frac{y - y_f}{y_0 - y_f}\right) = \frac{\ln\left(\frac{y - y_f}{y_0 - y_f}\right)}{\ln(r)} = \frac{\log\left(\frac{y - y_f}{y_0 - y_f}\right)}{\log(r)}$$

t	y
0	100
20	42
40	22
60	14
80	11
100	10





Endogenous Decrease is controlled from inside the system. There is feedback.

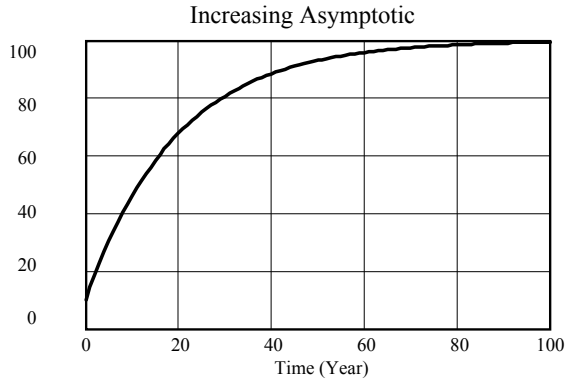
Increasing Asymptotic Exponential

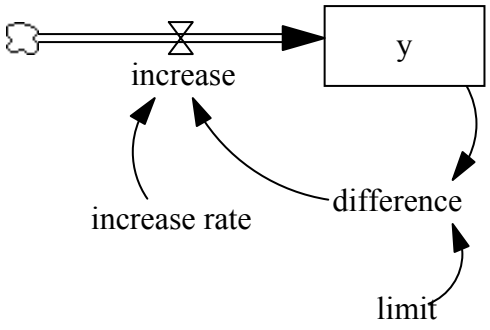
Multiply by a fractional constant repeatedly
The more you have, the less you get.
r is the common ratio.

$$y = y_0 + (y_f - y_0)(1 - (1 - r)^t) \quad r < 1$$

$$t = \log_{1-r}\left(1 - \frac{y - y_0}{y_f - y_0}\right) = \frac{\ln\left(1 - \frac{y - y_0}{y_f - y_0}\right)}{\ln(1 - r)} = \frac{\log\left(1 - \frac{y - y_0}{y_f - y_0}\right)}{\log(1 - r)}$$

t	y
0	10
20	68
40	88
60	96
80	98
100	99





Endogenous Increase is controlled from inside the system. There is feedback.

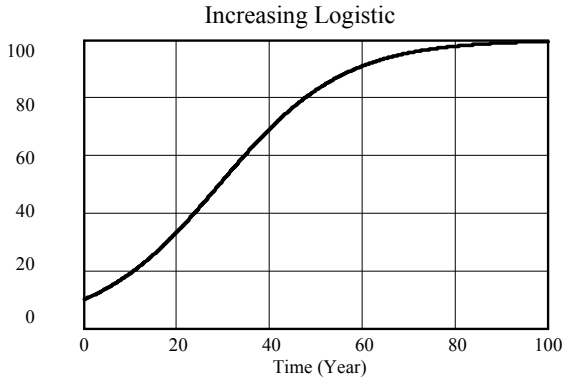
Increasing Logistic

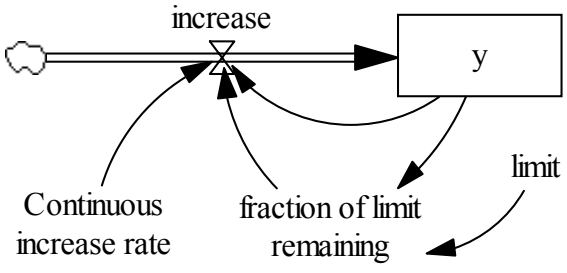
An Increasing Exponential becomes
An Increasing Asymptotic Exponential.

$$y = \frac{y_f}{1 + \frac{y_f - y_0}{y_0} e^{-rt}}$$

$$t = \frac{\ln\left(\frac{y_0(y_f - y)}{y(y_f - y_0)}\right)}{-r}$$

t	y
0	10
20	33
40	69
60	91
80	98
100	99





Endogenous Increase is controlled from inside the system. There is feedback.