The Logistic Equation

The Logistic Equation describes a circumstance where exponential growth is indicated but where that growth is approaching a fixed limit. Imagine a colony of bacteria growing in a jar. Bacteria multiply exponentially often doubling their numbers every hour. But since they are in a jar they can't continue to multiply like this forever. The maximum number the jar can hold is called the carrying capacity. Let's start with one bacteria and assume that the carrying capacity is 10000. First let's create an exponential model that ignores the carrying capacity. Let's model the bacteria growth for the first 48 hours. Put the equation $Y_1 = 2$^X in your calculator. Use the table function in your calculator to complete the table below. Then enter the logistic equation for the same situation into your calculator. It should look like this:

Y_2 = 10000/(1+(10000-1)/1*(1/2)^X) Use your table function to complete the rest of the table, then graph both equations using unique markings for each. Have your graph only go to 10000 bacteria, since that's all the jar can hold.

Time (hours)	bacteria (exponential model)	bacteria (logistic model)
0		
1		
2		
3		
4		
5		
6		
7		
10		
15		
20		
25		
26		
27		
28		
29		
30		
31		
32		

There are two forms of the exponential equation; $y = 2^x$ and $y = e^{.693x}$. These are two ways to write the same equation. Notice that .693 is the ln(2). The same idea is true for the logistic equation. Both these models assume that growth is continuous. The two forms are:

$$y = \frac{y_\infty}{1 + \frac{y_\infty - y_0}{y_0} e^{-rt}} \quad \text{and} \quad y = \frac{y_\infty}{1 + \frac{y_\infty - y_0}{y_0} (1/m)^t}$$

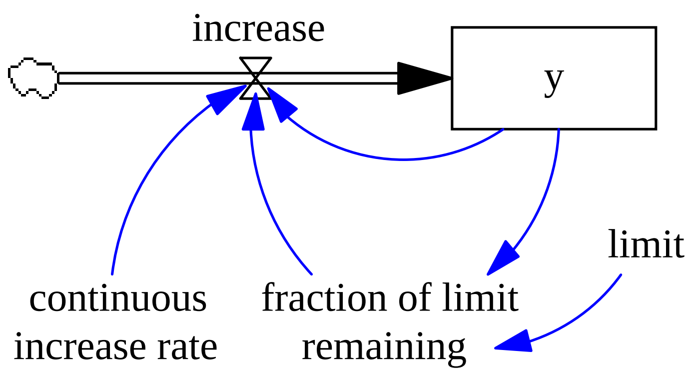
where y is the value of the dependent variable at any time, y_0 is the starting value, y_∞ is the carrying capacity or limit to growth, m is the multiplier (like the two in the previous example), t is time (or some other independent variable), and r is ln(m). You used the second form in your calculator on the previous page. Solving for t results in two solutions:

$$t = \frac{\ln\left(\frac{y_0(y_\infty - y)}{y(y_\infty - y_0)}\right)}{-r} \quad \text{and} \quad t = \frac{\ln\left(\frac{y_0(y_\infty - y)}{y(y_\infty - y_0)}\right)}{\ln\left(\frac{1}{m}\right)}$$

It may not be obvious that $-r = \ln(1/m)$, it's the same as saying that $r = \ln(m)$.

Now use both of these equation to find out when the jar is half full.

Here's the system dynamics model using $r = \ln(m)$ as the continuous growth rate.



Complete the diagram by adding the appropriate numbers.

Write a description of what you think is going on here.