Exponents and the Power of Exponential Growth

Take out a sheet of paper. Tear it into two equal halves. In the table below write the number of tears and the number of pieces of paper. Place one on top of the other to make a stack. Tear the stack into two equal halves. Make the two stacks into one big stack. In the table below write the number of tears and the number of pieces of paper in the stack. Tear the stack into two equal parts. Put the two stacks together to make one stack. In the table below write the number of tears and the number of pieces of paper in the stack. If you can continue this process for fifty tears bring the finished stack to your instructor and you can exchange it for $1000. If you give up on tearing, you can calculate the rest of the table with your calculator. Of course, then you don't get the $1000.

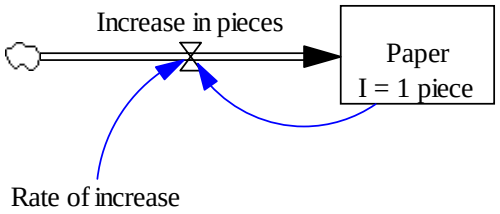
t # of tears	***p*** # of pieces	2^t
0	$1 = 2^0$	
1	$2 = 2^1$	
2	$4 = 2^2$	
3		
4		
5		
6		
7		
8		
9		
10		
11		
12		
13		
14		
15		
16		

t # of tears	***p*** # of pieces	2^t
17		
18		
19		
20		
21		
22		
23		
24		
25		
26		
27		
28		
29		
30		
31		
32		
33		

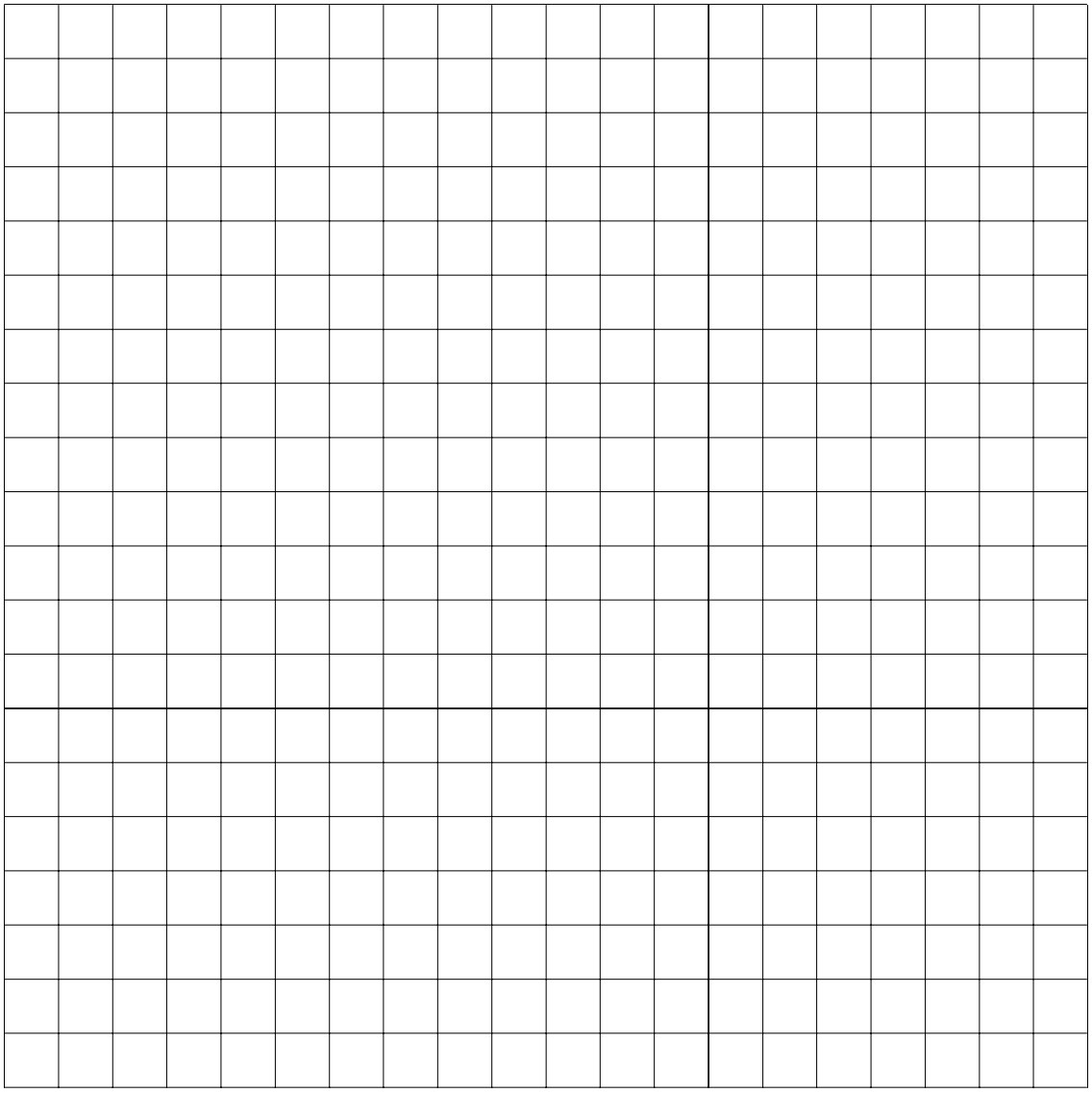
t # of tears	***p*** # of pieces	2^t
34		
35		
36		
37		
38		
39		
40		
41		
42		
43		
44		
45		
46		
47		
48		
49		
50		

If each piece is .003 of an inch thick, what is the height of your final stack?
In inches? _____ In feet? _____ In miles? _____

The stock-flow diagram has a feedback loop since the increase depends on the number of pieces.



Graph the results of your experiment. Graph the number of pieces as a function of the number of tears.



Your equation for this curve is number of pieces $= 2^{\text{number of tears}}$ or $p = 2^t$. To solve this equation for t means we want to know how many 2s we would have to multiply to make the number we are looking for. Let's say we want to know when we will have 64 pieces. How many 2 would we have to multiply to make 64? Sure, 6. There's a math shorthand way to ask this same question, it's called a logarithm or log. If I ask, "what is log base 2 of 32 ($\log_2 32$)." I am asking how many 2s need to be multiplied to make 32. Yep, 5. If I ask, "what is log base 10 of 1000000 ($\log_{10} 1000000$)." I am asking how many 10s need to be multiplied to make 1000000. Uh, huh, 6. If I ask, "what is log base 5 of 25 ($\log_5 25$)." I am asking how many 5s need to be multiplied to make 25. Yes, it's 2. So to solve our equation we simply take the log base 2 of both sides. So the left side now says $\log_2 p$ and the right side says $\log_2 2^t$. How many 2 need to be multiplied to make 2^t? Yes, t of them. So the solution to our equation is $t = \log_2 p$.